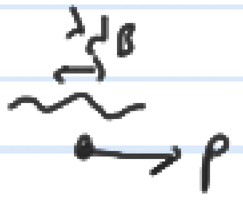
4/15/2020

· ZEEMAN SLOWING

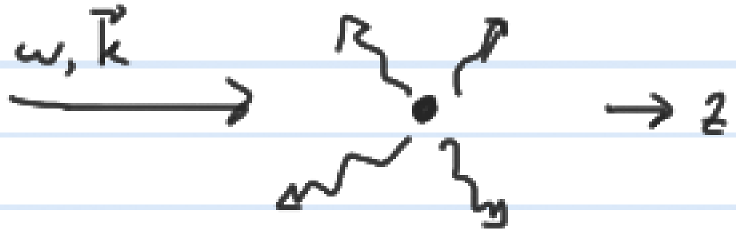
COLD ATOMS

de BROGLIE WAVE LENGTH $\lambda_{dB} = \frac{h}{p}$



COLD → SMALL p → LARGE λ_{dB} ⇒ CAN OBSERVE QUANTUM EFFECTS

SCATTERING FORCE $\vec{F}_{sc} = (\hbar\vec{k})(\Gamma \rho_{22}) = \frac{\hbar k \Gamma}{2} \frac{S}{1+S+(2\delta/\Gamma)^2}$

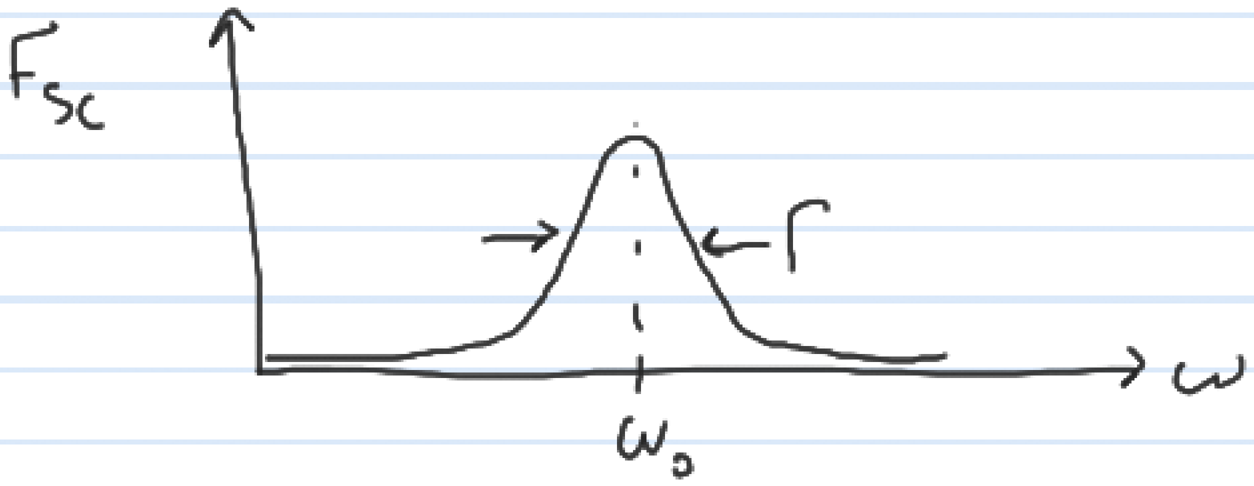


$S = I/I_{SAT}$, $\delta = \omega - \omega_0$

MAX. ACCELERATION ($s \to \infty$) $F_{sc}^{max} = \frac{\hbar k \Gamma}{2} \to a^{max} = \frac{F^{max}}{M} = \frac{\hbar k \Gamma}{2M}$

SODIUM: $\lambda = 589\,nm$, $M = 23u$, $\Gamma = 9.8 \times 2\pi \times MHz \to a^{max} = 9 \times 10^5\, m/s^2 \gg g$

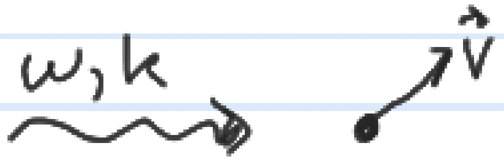
ON RESONANCE ($\delta = 0$): $a_S = \frac{S}{1+S}\, a^{max}$



$S \ll 1$: $F_{sc} \sim \frac{1}{1+\underbrace{(2\delta/\Gamma)^2}_{1}} = \frac{1}{2} \to \delta = \frac{\Gamma}{2}$

$S \gtrsim 1$: $\Gamma \to \Gamma\sqrt{1+S}$

FORCE ON MOVING ATOM ($\vec{k} = k\hat{z}$)

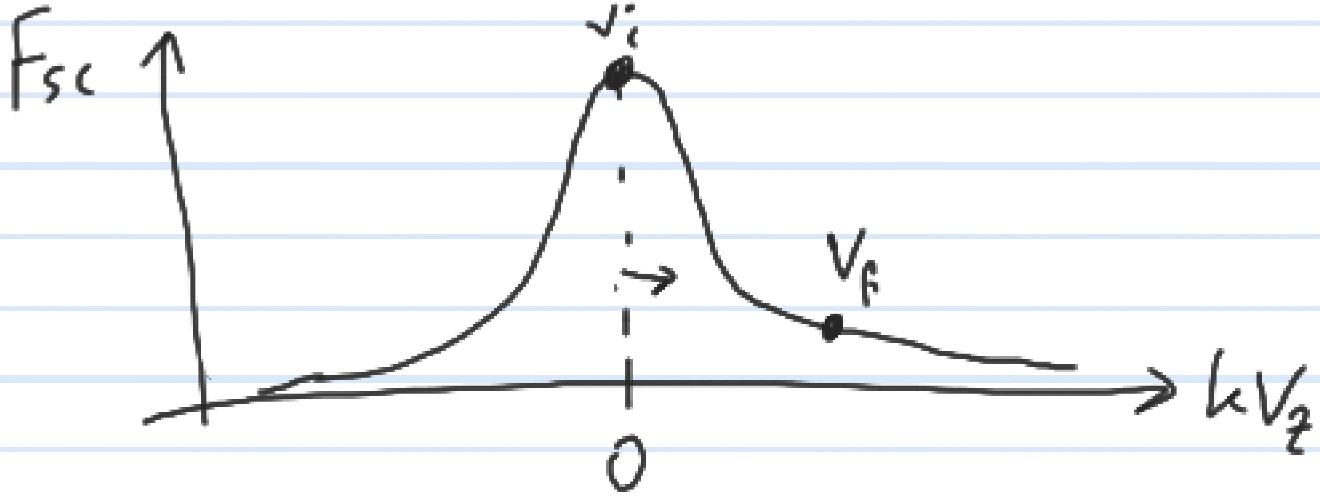


ATOM FRAME: $\omega' = \omega - kv_z$; RESONANCE: $\omega' = \omega_0 \Rightarrow \omega^* - kv_z = \omega_0 \to \omega^* = \omega_0 + kv_z$

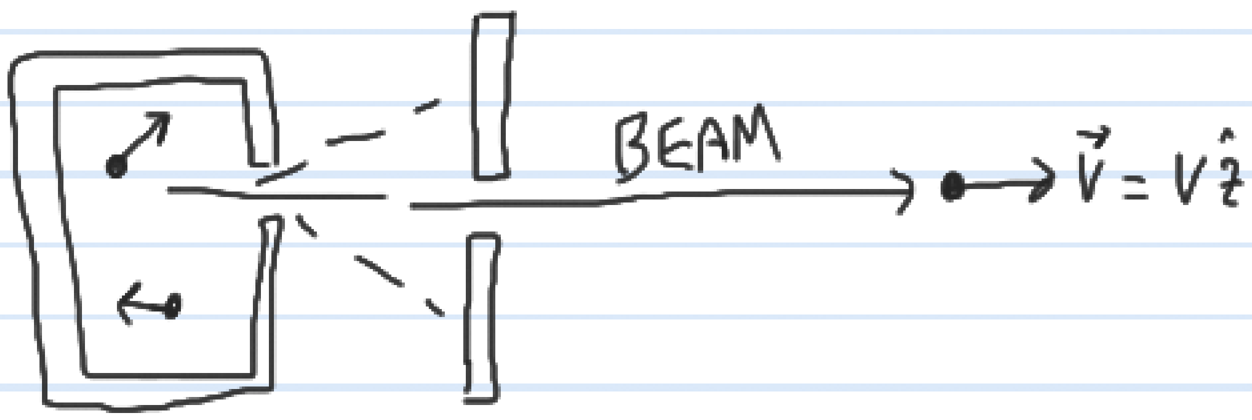
DETUNING: $\delta = \omega - \omega^* = \omega - \omega_0 - kv_z$

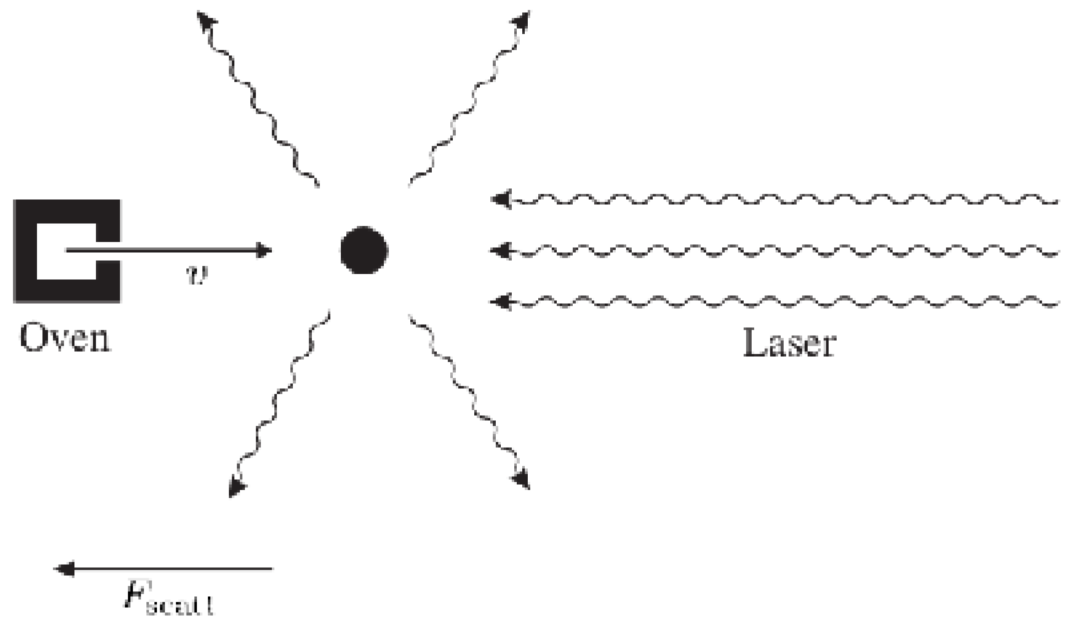
ACCELERATING AN ATOM

- ATOM INITIALY AT REST $\overset{\omega=\omega_0}{\rightsquigarrow} \bullet\, v=0$, SHINE RESONANT LIGHT
- ATOM ACCELERATES IN $\hat{z}$ $\underset{\omega'=\omega_0 - kv_z}{\rightsquigarrow} \bullet\!\!\rightarrow v$, NO LONGER RESONANT



SLOWING AN ATOMIC BEAM





PROBLEM: DOPPLER SHIFTED OUT OF RESONANCE

SOLUTION: MAKE ω_0 CHANGE TOO!

HOW? APPLY $\vec{B}$ THAT VARIES WITH z

(RECALL: ENERGY LEVELS DEPEND ON B, "ZEEMAN EFFECT")

USE STRETCHED STATES, ALKALI ATOM

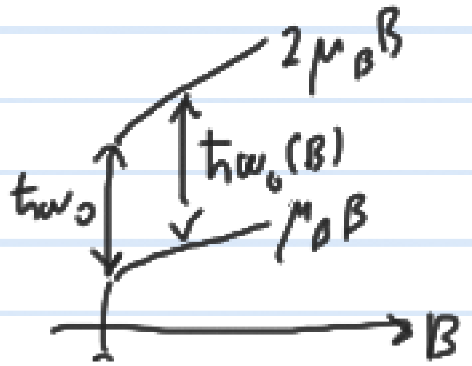
GROUND: $L=0$, $S=\frac{1}{2}$, $S_{1/2}$. STRETCHED(+): $S_z = \hbar S = \frac{\hbar}{2}$, $L_z = \hbar L = 0$, $I_z = \hbar I$

$$H' = \frac{\mu_B B}{\hbar}(g_L L_z + g_S S_z + g_I I_z) \; ; \quad g_L \simeq 1, \; g_S \simeq 2, \; g_I = 0$$

$$\Delta E_g = \mu_B B(0 + 2/2 + 0) = \mu_B B$$

EXCITED STATE: $L = 1$, $S = 1/2$; STRETCHED: $S_z = \frac{\hbar}{2}$, $L_z = \hbar$, $I_z = I\hbar$

$$\Delta E_e = \frac{\mu_B}{\hbar} B \left(g_L L_z + g_S S_z + g_I I_z \right) = \mu_B B \left(1 + 2/2 + 0 \right) = 2\mu_B B$$



$$E = \hbar\omega$$

$$\frac{E}{\hbar} = \omega$$

$$\Delta(E_e - E_g) = 2\mu_B B - \mu_B B = \mu_B B$$

RESONANCE CONDITION: $\overset{v}{\longrightarrow}$ $\quad \xleftarrow{-k,\ \omega}$

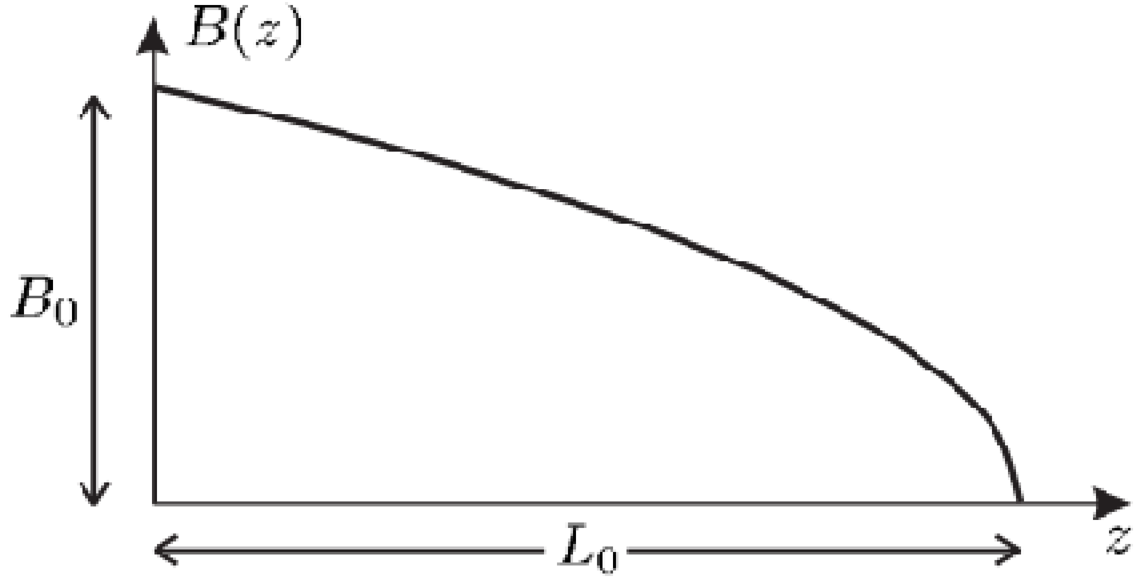
$$\omega' = \omega_0(B) \longrightarrow \omega + kV = \omega_0 + \mu_B B/\hbar$$

$$\to \frac{\mu_B}{\hbar} B(z) = \omega - \omega_0 + kV(z)$$

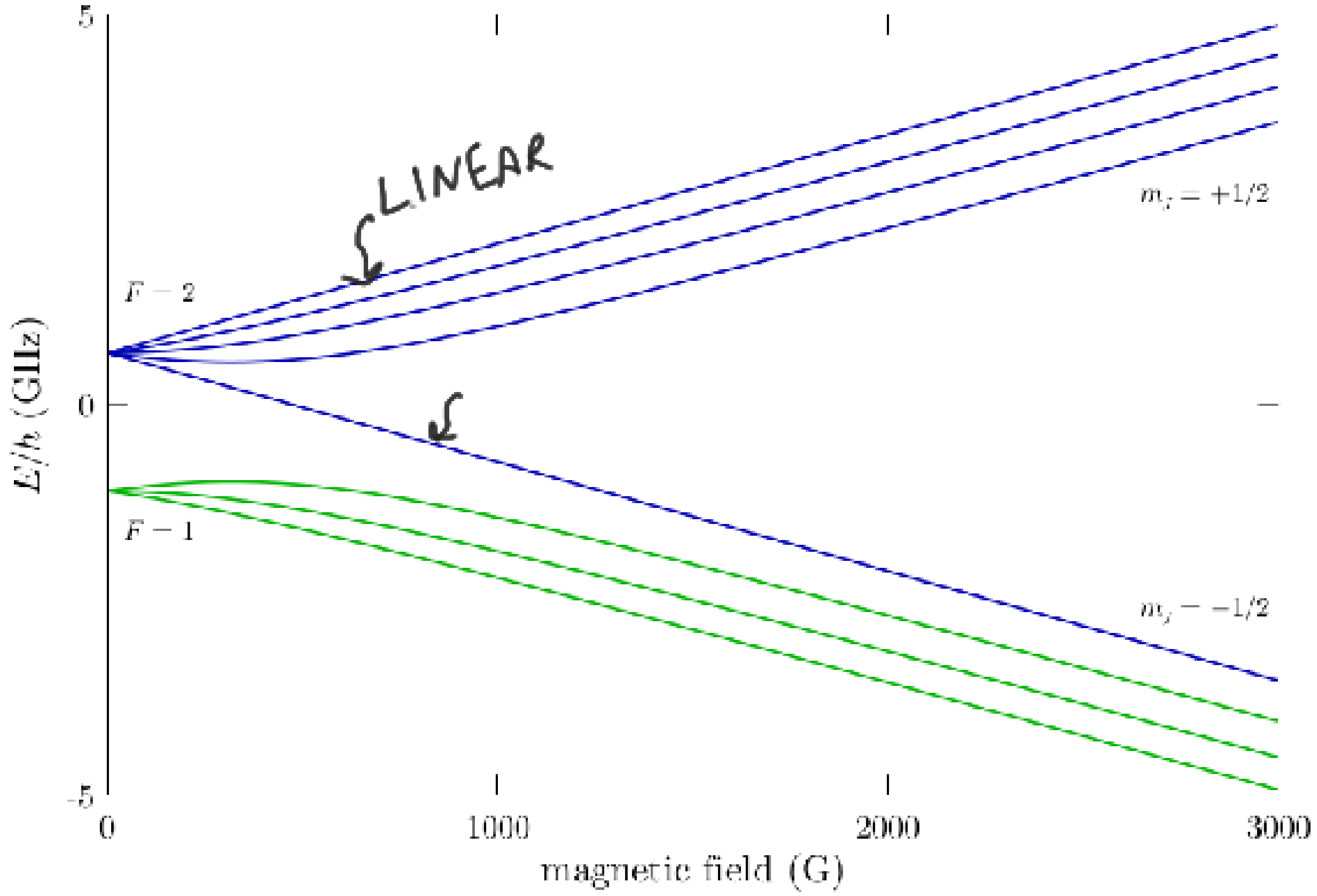
$V(z) = ?$ ASSUME CONST. $a = -\eta a^{max} = -\frac{S}{1+S} a^{max}$

$$V(z) = \sqrt{V_0^2 + 2az} = \sqrt{V_0^2 - 2\eta a^{max} z}$$

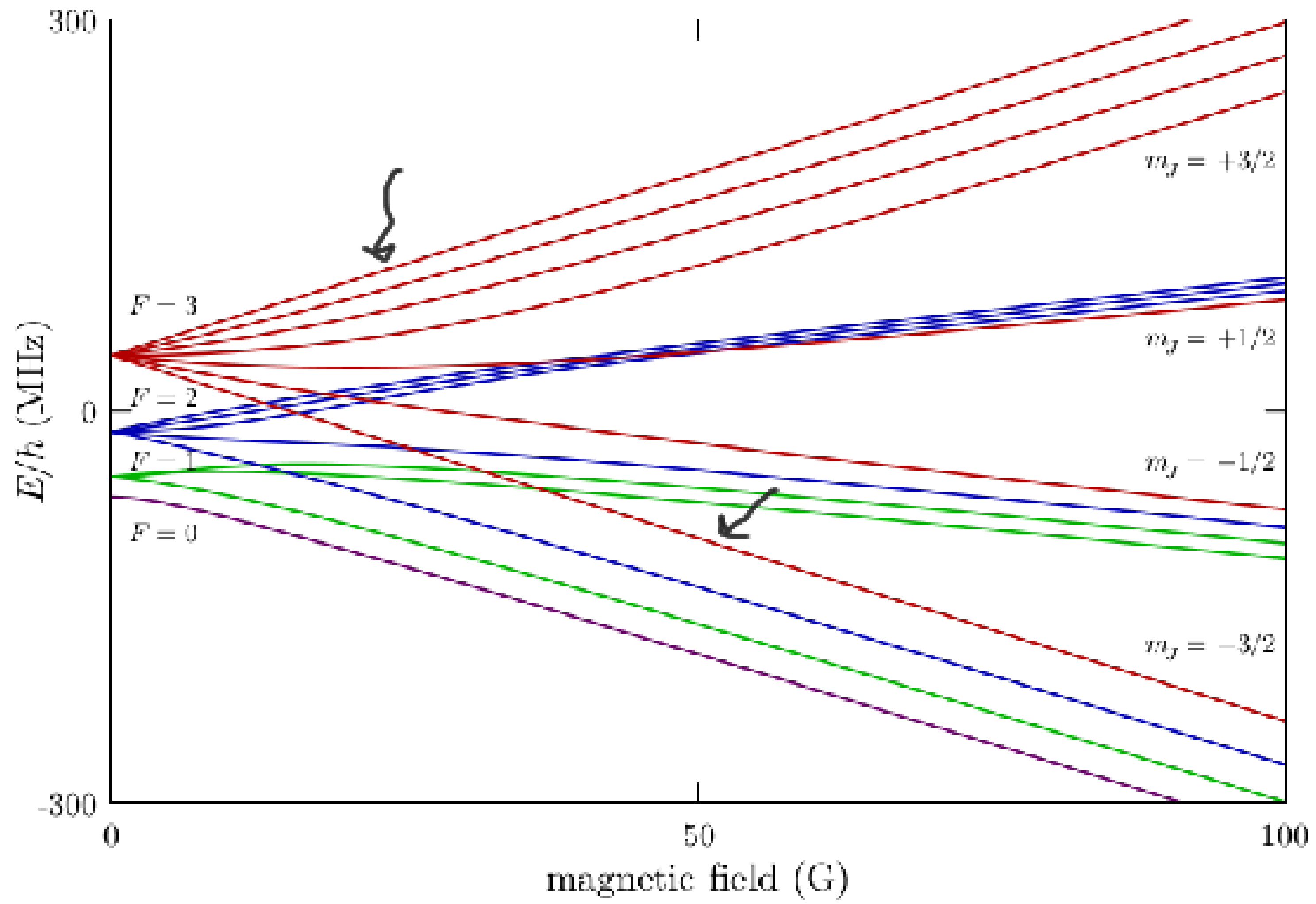
IDEAL $B(z) = \frac{\hbar}{\mu_B} (\omega - \omega_0) + \frac{\hbar}{\mu_B} k \sqrt{V_0^2 - 2\eta a^{max} z}$



sodium-23 ground state



sodium-23 P_3/2 excited state

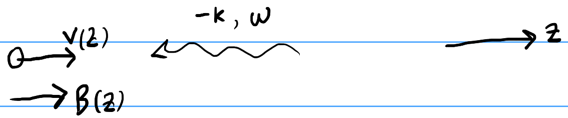


PHY 446 SPRING 2020 LECTURE 24

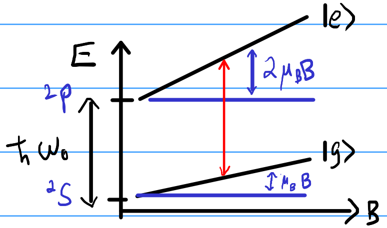
4/20/2020

- OPTICAL MOLASSES

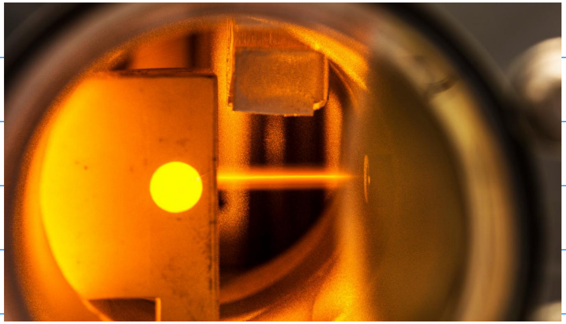
LAST TIME: ZEEMAN SLOWING TO SLOW A BEAM OF ATOMS



STRETCHED STATES:



$E_e - E_g = \hbar\omega_0 + \mu_B B$



sodium beam, Gretchen Cambpell, UMD

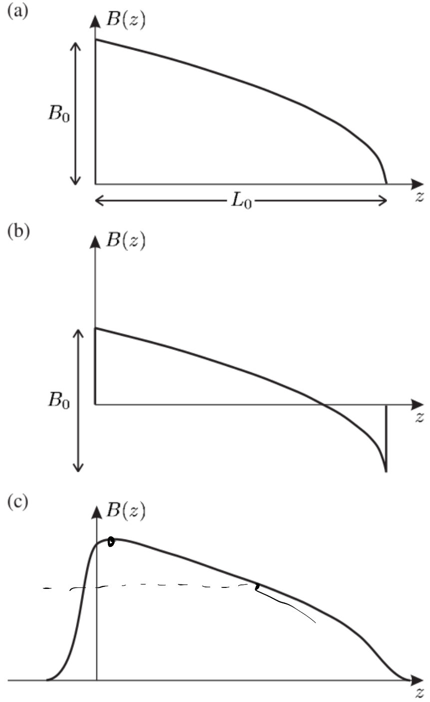
RESONANCE: $\omega + kv = \omega_0 + \mu_B B/\hbar$

- B(z) VARIES TO COMPENSATE DOPPLER SHIFT

MAGNETIC FIELD PROFILE

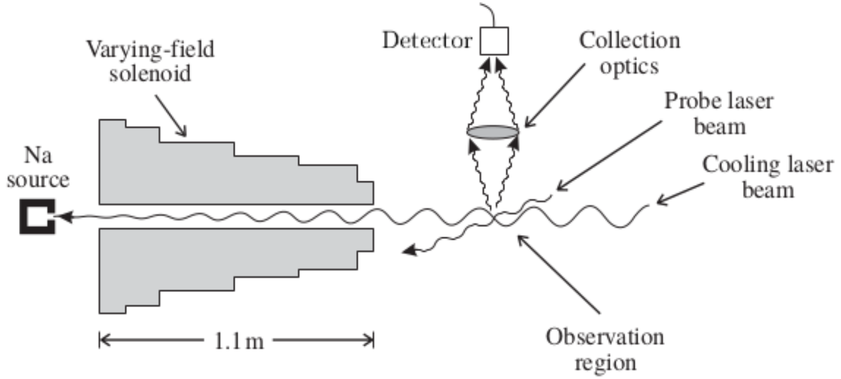
$$B(z) = \frac{\hbar}{\mu_B}(\omega-\omega_0) + \frac{\hbar k}{\mu_B} v(z)$$

CONSTANT ACCELERATION $a<0$: $v(z) = \sqrt{v_0^2 + 2az}$

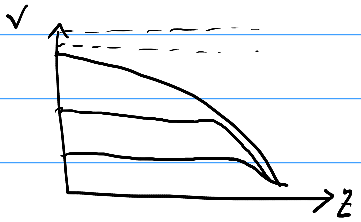


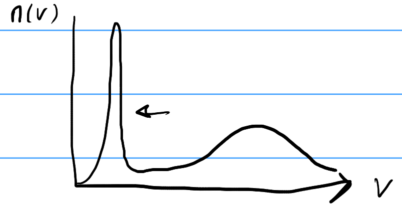
- ARBITRARY OFFSET DEPENDS ON ω

Foot, Fig. 9.3

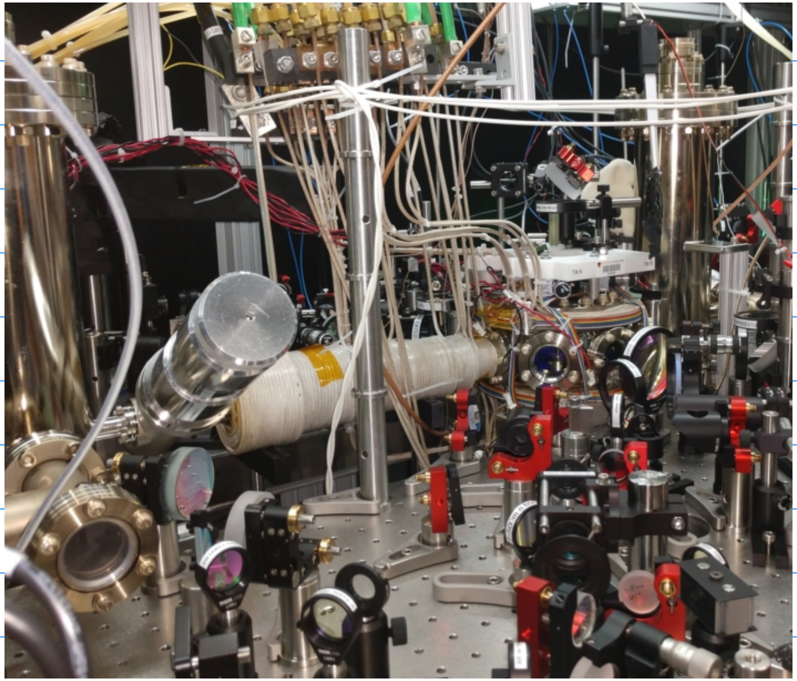


Foot, Fig. 9.2

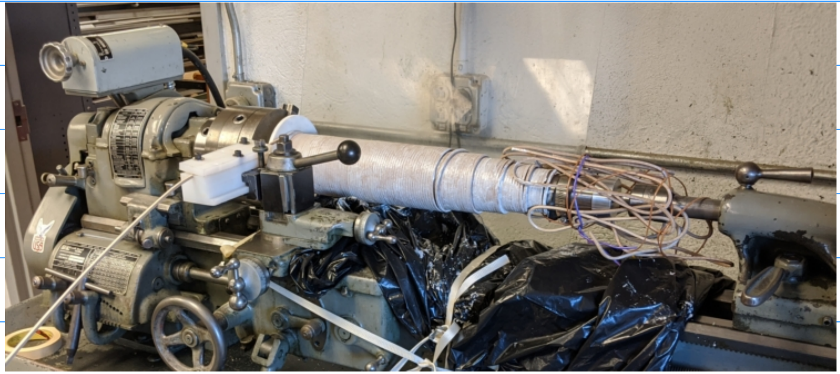


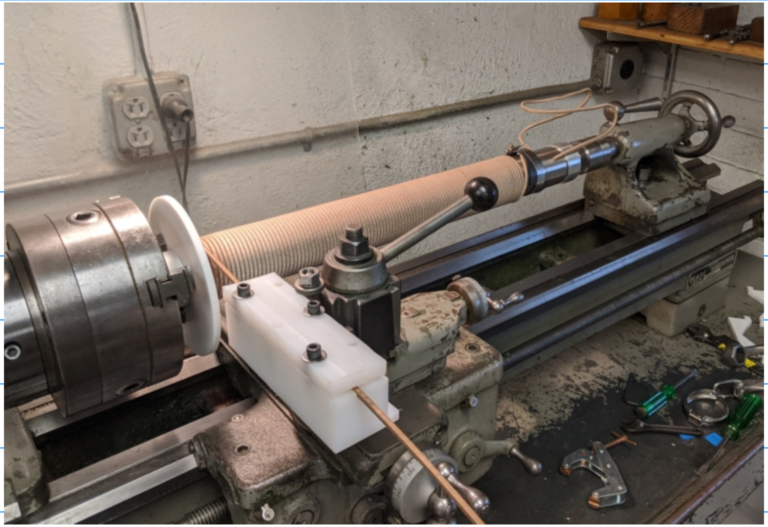


Actual Zeeman Slowers



cold atom setup, Waseem Bakr's group, Princeton

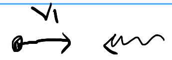


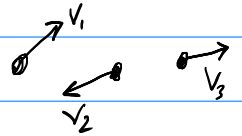


Sommer Lab Zeeman slower under construction, Lehigh!

OPTICAL MOLASSES

Molasses, Wikipedia

- KNOW HOW TO SLOW A BEAM 
- HOW CAN WE COOL A GAS?



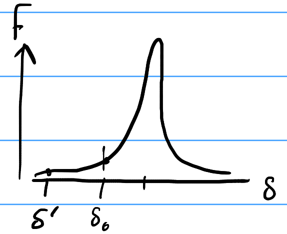
→ ADD FRICTION FORCE $\vec{F} \approx -\alpha \vec{v}$

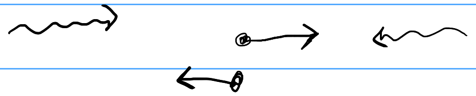
CONSIDER ATOM MOVING RIGHT 

- NEED LIGHT TRAVELING LEFT ; RED DETUNED

NOW, ATOM MOVING LEFT 

- NEED LIGHT TRAVELING RIGHT ; RED DETUNED





SLOWED IN BOTH DIRECTIONS!

"MOLASSES"

FORCE IN OPTICAL MOLASSES (1D)

ONE BEAM:

$\overset{\rightsquigarrow}{k,\omega} \qquad o \rightarrow v$

- $B = 0$

LET $\delta = \omega - \omega_0$

DETUNING IN ATOM FRAME: $\delta' = \omega - \omega_0 - kv = \delta - kv$

SUPPOSE $s \ll 1$ $\quad (s = I/I_{SAT})$

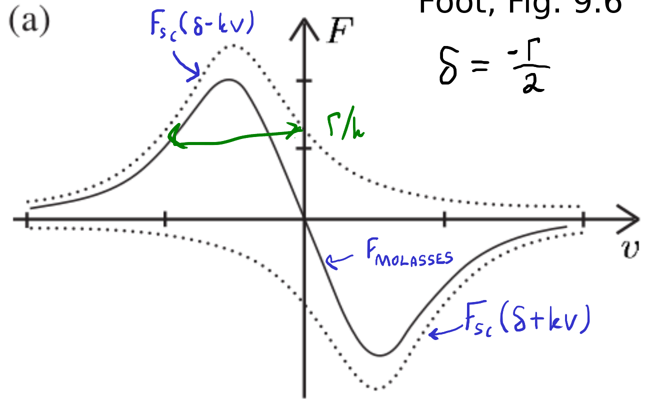
SCATTERING FORCE: $F_{sc}(\delta') = \hbar k \Gamma \rho_{22}(\delta') = \hbar k \Gamma \, \frac{1}{2} \, \frac{s}{1 + \cancel{s} + (2\delta'/\Gamma)^2}$

$$\simeq \frac{\hbar k \Gamma}{2} \, \frac{s}{1 + [2(\delta - kv)/\Gamma]^2}$$

TWO BEAMS: $\overset{k,\omega}{\rightsquigarrow} \quad o \rightarrow v \quad \overset{-k,\omega}{\leftsquigarrow}$

- FOR $s \ll 1$, FORCES ADD

$$F_{MOLASSES} = F_{sc}(\delta - kv) - F_{sc}(\delta + kv)$$



Foot, Fig. 9.6

LINEAR SLOPE FOR SMALL v

FOR $kv \ll \Gamma$, TAYLOR EXPAND IN kv

$$F_{MOLASSES} = F_{sc}(\delta - kv) - F_{sc}(\delta + kv) \approx \Big[\underbrace{\cancel{F_{sc}(\delta)}}_{v=0} - \frac{dF_{sc}}{d\delta}(\delta)\, kv\Big] - \Big[\cancel{F_{sc}(\delta)} + \frac{dF_{sc}}{d\delta}(\delta)\, kv\Big]$$

$$= -2\frac{dF_{sc}}{d\delta} kv \qquad \equiv -\alpha v$$

FRICTION COEFFICIENT $\alpha = 2k\,\frac{dF_{sc}}{d\delta}$, WHERE $F_{sc}(\delta) = \frac{\hbar k \Gamma}{2}\frac{s}{1+(2\delta/\Gamma)^2}$

$$\alpha = \cancel{2}k\left(\frac{\hbar k\Gamma}{\cancel{2}}\right) s\, \underbrace{\frac{d}{d\delta}\left\{\left[1+(2\delta/\Gamma)^2\right]^{-1}\right\}}_{-\left[1+(2\delta/\Gamma)^2\right]^{-2}\left(\frac{2}{\Gamma}\right)^2(2\delta) = \frac{-8\delta/\Gamma^2}{\left[1+(2\delta/\Gamma)^2\right]^2}}$$

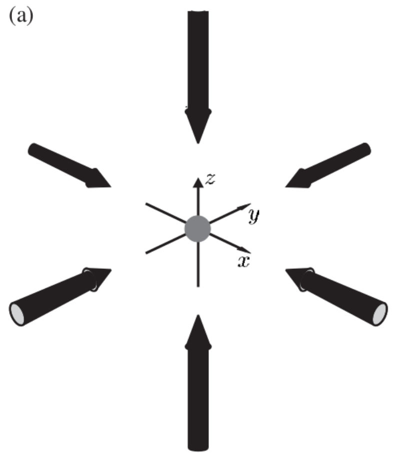
$$= \hbar k^2 s\,\frac{-8\delta/\Gamma}{\left[1+(2\delta/\Gamma)^2\right]^2} = 4\hbar k^2 s\,\frac{-2\delta/\Gamma}{\left[1+(2\delta/\Gamma)^2\right]^2}$$

3D OPTICAL MOLASSES

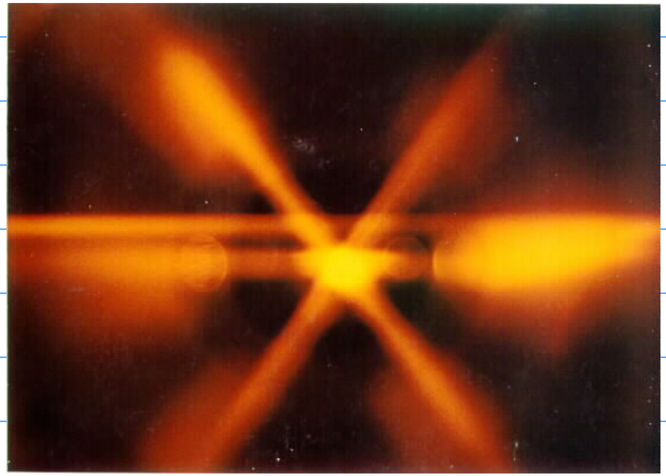
- SIX BEAMS: $\pm x$, $\pm y$, $\pm z$

FOR $kv \ll \Gamma$, $s \ll 1$:

$$\vec{F}_{\text{MOLASSES}} = (-\alpha v_x, -\alpha v_y, -\alpha v_z) = -\alpha \vec{v}$$



Foot, Fig. 9.5



NIST, sodium molasses

First molasses:
Mg+ and Ba+ ions, 1978

Neutral atom molasses:
sodium, 1985, Chu et al, Bell Labs

COOLING

$$E = \frac{1}{2}Mv^2$$

$$\frac{dE}{dt} = M\vec{v}\cdot\frac{d\vec{v}}{dt} = \vec{v}\cdot\vec{F}_{MOLASSES}$$

$$\approx -\vec{v}\cdot(\alpha\vec{v}) = -\alpha v^2$$

$$= -\frac{2\alpha}{M}E = -\frac{1}{\tau}E$$

$$\Rightarrow E(t) = E_0 e^{-t/\tau}$$

$$\tau = M/(2\alpha)$$

$\Rightarrow E \to 0$ AS $t \to \infty$

- NOT QUITE TRUE
- SPONTANEOUS EMISSION
  - → LOWER LIMIT

DOPPLER LIMIT

- SPONTANEOUSLY EMITTED PHOTONS CAUSE RECOIL

$$M\Delta\vec{v} = -\hbar\vec{k} \Rightarrow \Delta\vec{v} = -\hbar\vec{k}/M$$

- RANDOM WALK OF VELOCITY $\vec{v}$

INCREMENT: $v_r = \hbar k/m$ (RECOIL VELOCITY)

RANDOM WALK (1D): $\langle v_z^2\rangle_{spont.} = N\cdot v_r^2/3$

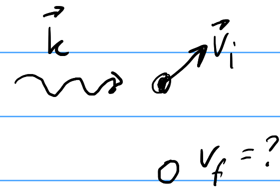
$$N = R_{sc}t = \Gamma\rho_{22}\,t$$

- RANDOMNESS IN ABSORPTION: $\langle v_z^2\rangle_{abs.} = Nv_r^2 = v_r^2 R_{sc}t$

PHY 446 LECTURE 25 4/22/2020

- MOLASSES TEMPERATURE, DOPPLER LIMIT
- MAGNETO-OPTICAL TRAP (MOT)

QUICK WARM-UP



- ATOM, INITIAL VELOCITY $\vec{v}_i$ MASS M
- ABSORBS A PHOTON OF WAVEVECTOR $\vec{k}$

a) FIND VELOCITY $\vec{v}_f$ OF ATOM AFTER ABSORBING

b) FIND RECOIL VELOCITY $v_r = |\vec{v}_f - \vec{v}_i|$

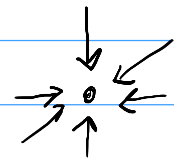
a) PHOTON MOMENTUM $\hbar\vec{k}$, TOTAL $\vec{P}_i = M\vec{v}_i + \hbar\vec{k} = \vec{P}_f = M\vec{v}_f$

$$\Rightarrow \vec{v}_f = \vec{v}_i + \hbar\vec{k}/M$$

b) $\vec{v}_f - \vec{v}_i = \hbar\vec{k}/M$, $\boxed{v_r = \hbar k/M}$

OPTICAL MOLASSES – TEMPERATURE LIMIT

6 RED-DETUNED BEAMS



- AVG FORCE $\vec{F} \approx -\alpha \vec{v}$, FOR $v \ll \Gamma/k \rightarrow$ COOLING
- FLUCTUATIONS IN $\vec{F}$ CAUSE HEATING
- WHAT IS THE EQUILIBRIUM TEMPERATURE?

PRELIMINARIES:

ASSUME LOW INTENSITY ($s \ll 1$)

SCATTERING RATE PER BEAM: $R_{sc} = \Gamma \rho_{22}|_{\text{ONE BEAM}} \approx \Gamma \frac{s/2}{1+[2\delta/\Gamma]^2}$

DAMPING COEF. $\alpha = 4\hbar k^2 s \frac{-2\delta/\Gamma}{[1+(2\delta/\Gamma)^2]^2}$

RECOIL VELOCITY $v_r = \hbar k/M$

EXPRESS α IN TERMS OF R_{sc} AND v_r:

$$\alpha = \frac{8}{\hbar}(M v_r)^2 \frac{R_{sc}}{\Gamma} \frac{1}{g(-2\delta/\Gamma)} \qquad (*)$$

WHERE $g(x) = \dfrac{1+x^2}{x}$, $x = -2\delta/\Gamma$

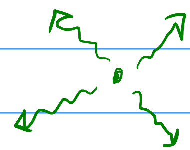
PHOTON SCATTERING - TWO PARTS

- ABSORB PHOTON : $\Delta\vec{v} = \vec{v}_{abs}$
- EMIT PHOTON : $\Delta\vec{v} = \vec{v}_{emit}$

ATOM VELOCITY : $\vec{v}_f = \vec{v}_i + \vec{v}_{abs} + \vec{v}_{emit}$

MAGNITUDES : $|\vec{v}_{abs}| = |\vec{v}_{emit}| = v_r$

$$\langle M \vec{v}_{emit} \rangle = -\langle \hbar \vec{k}_{emit} \rangle$$



SPONT. EMISSION : $\langle \vec{v}_{emit} \rangle =$ $\underline{\;0\;}$

MEAN FORCE GIVES $\langle \vec{V}_{abs} \rangle$:

$$\langle \vec{F} \rangle = \vec{F}_{MOLASSES} \approx -\alpha \vec{V}$$

ALSO:

$$\langle \vec{F} \rangle = M \left\langle \frac{d\vec{V}}{dt} \right\rangle = M \langle V_{abs} + V_{emit} \rangle \left(6\ R_{sc} \right) = M \langle \vec{V}_{abs} \rangle \left(6 R_{sc} \right)$$

$$\text{GIVES: } \langle \vec{V}_{abs} \rangle = \frac{\langle \vec{F} \rangle}{6 M R_{sc}} = \frac{-\alpha \vec{V}}{6 M R_{sc}}$$

CHANGE IN v^2 :

$$\vec{v}_f = \vec{v}_i + \left(\vec{V}_{abs} + \vec{V}_{emit} \right)$$

$$v_f^2 = \vec{v}_f \cdot \vec{v}_f = v_i^2 + 2 \vec{v}_i \cdot \left(\vec{V}_{abs} + \vec{V}_{emit} \right) + \left(\vec{V}_{abs} + \vec{V}_{emit} \right)^2$$

AVG VALUE :

$$\langle v_f^2 - v_i^2 \rangle = 2 \vec{v}_i \cdot \Big(\underbrace{\langle \vec{V}_{abs} \rangle}_{\text{KNOW}} + \cancelto{0}{\langle \vec{V}_{emit} \rangle} \Big) + \underbrace{\langle V_{abs}^2 \rangle}_{v_r^2} + \underbrace{\langle V_{emit}^2 \rangle}_{v_r^2} + 2 \cancelto{0}{\langle \vec{V}_{abs} \rangle \cdot \langle \vec{V}_{emit} \rangle}$$

$$\langle v_f^2 - v_i^2 \rangle = 2\vec{v}_i \cdot \left(\frac{-\alpha}{6MR_{sc}}\right)\vec{v}_i + 2v_r^2$$

AVG RATE:

$$\frac{d}{dt}\langle v^2 \rangle = \langle v_f^2 - v_i^2 \rangle (6R_{sc}) = \underbrace{-\left(\frac{2\alpha}{M}\right)v^2}_{\text{COOLING}} + \underbrace{12R_{sc}v_r^2}_{\text{HEATING}}$$

STEADY-STATE: $d\langle v^2 \rangle / dt = 0$

$$\Rightarrow \left(\frac{2\alpha}{M}\right)v^2 = 12R_{sc}v_r^2$$

$$v^2 = \frac{6R_{sc}M}{\alpha}v_r^2$$

USE:

$$\alpha = \frac{8}{\hbar}(Mv_r)^2\frac{R_{sc}}{\Gamma}\frac{1}{g(-2\delta/\Gamma)}$$

$$\frac{R_{sc}}{\alpha} = \frac{\hbar\Gamma}{8(Mv_r)^2}g(-2\delta/\Gamma)$$

$$\langle v^2 \rangle = (6Mv_r^2)\frac{\hbar\Gamma}{8(Mv_r)^2}g(-2\delta/\Gamma)$$

KINETIC ENERGY: $\langle \frac{1}{2}Mv^2 \rangle = \frac{3}{8}\hbar\Gamma\, g(-2\delta/\Gamma)$

TEMPERATURE: USE EQUIPARTITION THEOREM (CLASSICAL STAT. MECH.)

GENERAL: EACH "QUADRATIC DEGREE OF FREEDOM" HAS $\frac{1}{2}k_BT$ AVG. ENERGY

$$\langle \tfrac{1}{2}Mv^2 \rangle = \tfrac{1}{2}M\langle \underbrace{v_x^2 + v_y^2 + v_z^2}_{3\text{ D.O.F.}} \rangle = \tfrac{3}{2}k_BT$$

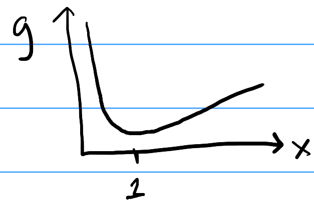
SOLVE FOR k_BT:

$$\tfrac{3}{2}k_BT = \tfrac{1}{2}Mv^2 = \tfrac{3}{8}\hbar\Gamma\, g(-2\delta/\Gamma)$$

$$\boxed{k_BT = \frac{\hbar\Gamma}{4}\, g(-2\delta/\Gamma)}$$

WHERE $g(x) = \dfrac{1+x^2}{x} = \dfrac{1}{x} + x$

LOWER LIMIT:



MIN: $g(1) = 2$

$$k_B T = \frac{\hbar\Gamma}{4} g(-2\delta/\Gamma)$$

MINIMUM TEMPERATURE: $\delta = ?$ $\quad x = \frac{-2\delta}{\Gamma} = 1 \Rightarrow \delta = -\Gamma/2 \quad (\delta = \omega - \omega_0)$

DOPPLER LIMIT:

$$k_B T_D = \frac{\hbar\Gamma}{4} 2 = \frac{\hbar\Gamma}{2}$$

E.X. ^{23}Na $(\Gamma = 2\pi \times 9.8\ MHz)$

a) FIND DOPPLER LIMIT $T_D = \frac{\hbar\Gamma}{2k_B} = 235\,\mu K$

b) FIND T AT $\delta = -\Gamma$

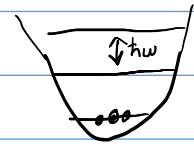
MAGNETO-OPTICAL TRAP (MOT)

- MOLASSES COOLS, BUT DOES NOT CONFINE
- CONFINEMENT (TRAPPING) $\rightarrow$ HIGHER ATOM DENSITY

MOTIVATION: INTERATOMIC DISTANCE d

DE BROGLIE WAVELENGTH λ_{dB}

$d \lesssim \lambda_{dB} \Rightarrow$ QUANTUM GAS



BOSONS: BOSE-EINSTEIN CONDENSATE

FERMIONS: DEGENERATE FERMI GAS $\rightarrow$ MODEL OF METAL

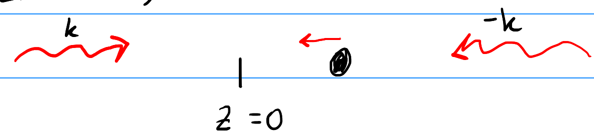
SMALL d REQUIRES LARGE DENSITY $n_a = 1/d^3$

MEAN FORCE: $F_{MOT}^{(z)} \approx \underbrace{-\alpha v_z}_{\text{COOLING}} \underbrace{- A_z z}_{\text{TRAPPING}}$ (SIMILAR FOR x AND y)

HOW? USE $\vec{B}$ FIELD GRADIENT

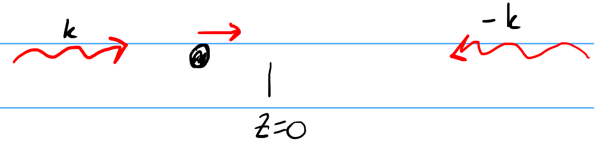
MOT : GENERAL IDEA (1D)

FOR $z > 0$:



INTERACTS WITH $-k$ BEAM

FOR $z < 0$:

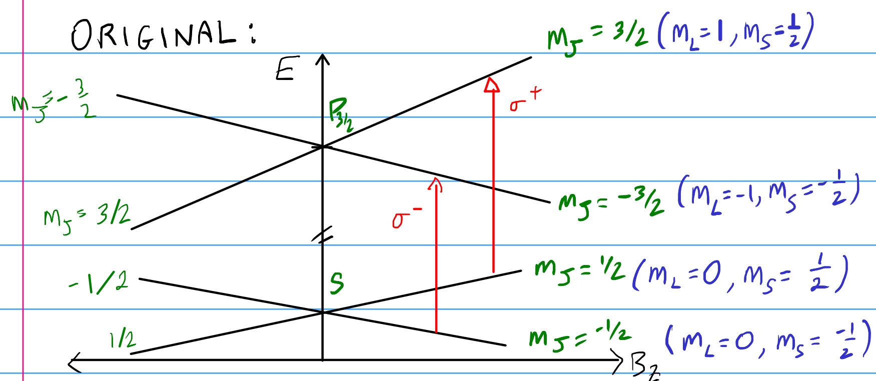


INTERACTS WITH $+k$ BEAM

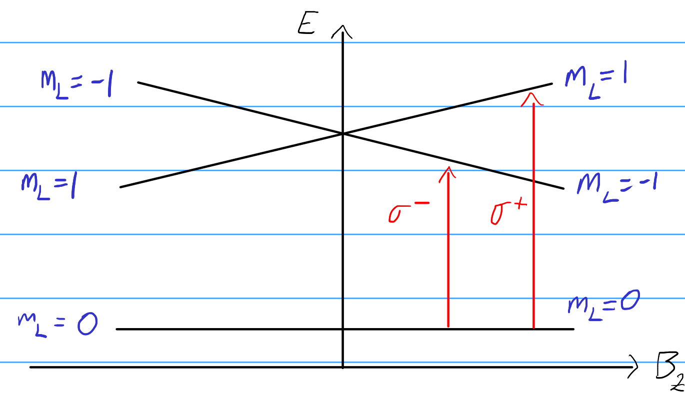
DETAILS: FIRST INTRODUCE SIMPLIFIED ENERGY LEVEL DIAGRAMS

- CONSIDER STRETCHED STATES
- SUBTRACT $-\vec{\mu}_s \cdot \vec{B}$ FROM ENERGIES (BECAUSE m_s DOESN'T CHANGE, ONLY m_L)

ORIGINAL:



$\Rightarrow$



PHY 446 LECTURE 26

4/27/2020

- MAGNETO-OPTICAL TRAPS (MOT)
- OPTICAL DIPOLE TRAPS (ODT)

MOT

MEAN FORCE: $F^{(z)}_{MOT} \approx \underbrace{-\alpha v_z}_{\text{COOLING}} \underbrace{- A_z z}_{\text{TRAPPING}}$ (SIMILAR FOR x AND y)

HOW?

USE RED-DETUNED BEAMS FOR DOPPLER COOLING

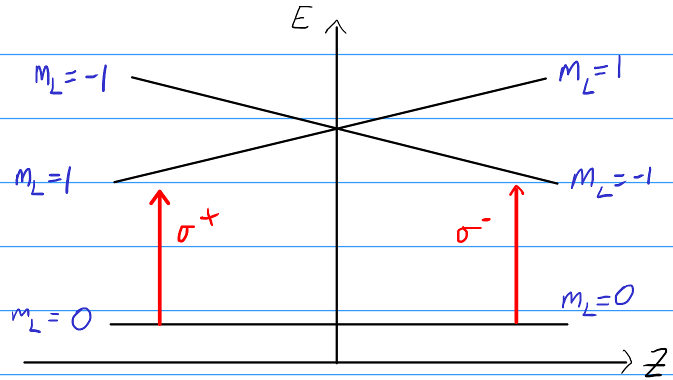
USE $\vec{B}$ FIELD GRADIENT FOR POSITION DEPENDENCE

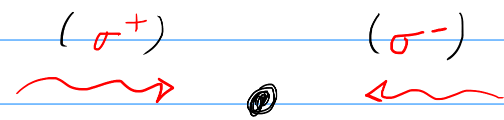
HOW TO TRAP & COOL: SOLUTION

- APPLY B-FIELD GRADIENT : $B_z = B' z$
- USE RED DETUNED BEAMS, ONE σ^+, ONE σ^-.

WHICH ONE SHOULD GO LEFT/RIGHT?

LEVELS vs. POSITION:



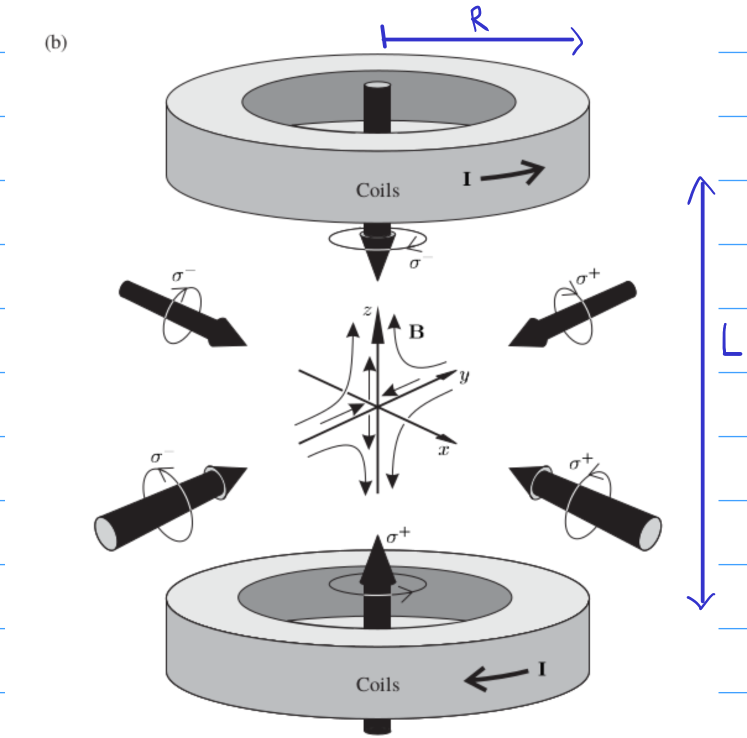


ATOM $z > 0 \Rightarrow$ INTERACTS MORE WITH $\sigma^- \Rightarrow \sigma^-$ SHOULD TRAVEL <u>LEFT</u>

ATOM $z < 0 \Rightarrow$ INTERACTS MORE WITH $\sigma^+ \Rightarrow \sigma^+$ SHOULD TRAVEL <u>RIGHT</u>

MOT - 3D

- USE 3D GRADIENT IN $\vec{B}$



FOOT, FIG. 9.9

$\vec{B}(x,y,z) \approx (ax, by, B'z)$ FOR $|\vec{r}| \ll R, L$

MAXWELL: $\vec{\nabla}\cdot\vec{B} = 0$

$\Rightarrow a + b + B' = 0$

CYL. SYMMETRY: $\partial B_x / \partial x = \partial B_y / \partial B_y$

$\Rightarrow a = b$

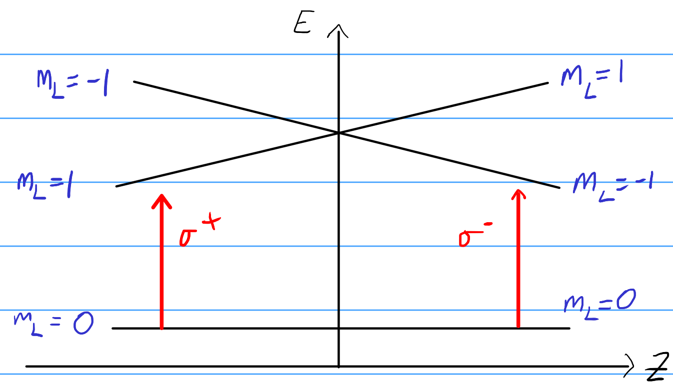
COMBINE:

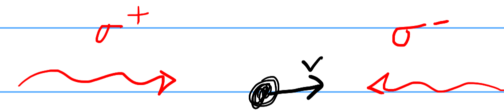
$2a + B' = 0$

$a = -\frac{1}{2} B'$

$\vec{B}(x,y,z) \approx \frac{1}{2} B' (-1, -1, 2) \cdot \vec{r}$

FORCE IN MOT (1D)





EFFECTIVE DETUNING: LET $\delta = \omega - \omega_0 < 0$

σ^+ BEAM: $\delta_+ = \delta - kv - \mu_B B' z/\hbar$

σ^- BEAM: $\delta_- = \delta + kv + \mu_B B' z/\hbar$

$$F_{MOT}^{(z)} = F_{sc}(\delta_+) - F_{sc}(\delta_-) \qquad (\text{FOR } S \ll 1)$$

WHERE $F_{sc}(\delta') = \hbar k \Gamma \rho_{22}(\delta')\Big|_{\text{ONE BEAM}} = \dfrac{\hbar k \Gamma}{2} \dfrac{s}{1 + (2\delta'/\Gamma)^2}$

FORCE IN MOT (1D) CONTINUED

$$F_{MOT}^{(z)} = F_{sc}(\delta - kv - \beta z) - F_{sc}(\delta + kv + \beta z)$$

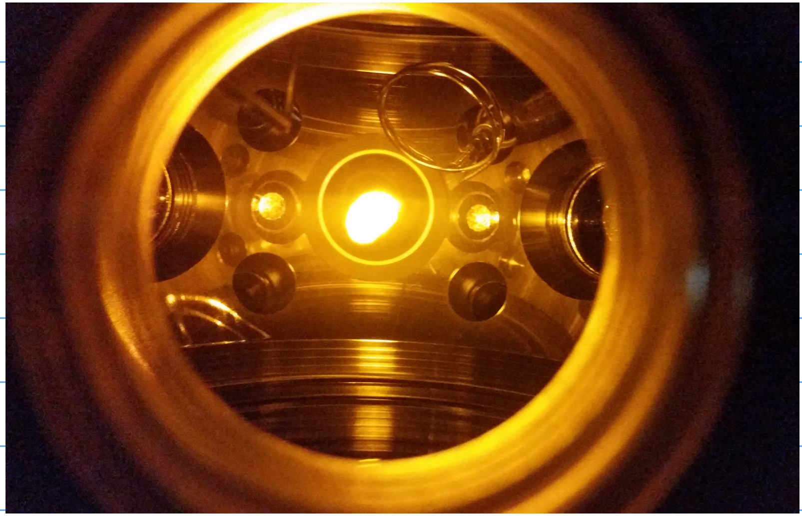
WHERE $\beta = \frac{\mu_B}{\hbar} B'$

TAYLOR EXPAND FOR $|kv + \beta z| \ll \Gamma$

$$F_{MOT}^{(z)} \approx -2 \frac{dF_{sc}}{d\delta}(\delta)(kv + \beta z) \equiv -\alpha v - A z$$

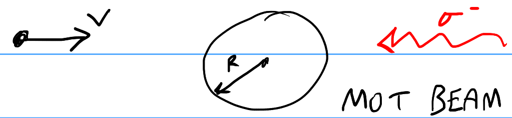
$\alpha = 2k \frac{dF_{sc}}{d\delta}$ ← SAME AS MOLASSES

$A = 2\beta \frac{dF_{sc}}{d\delta}$ ← SPRING CONSTANT

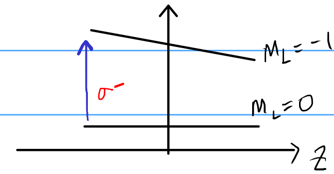


Sodium MOT
Schwettmann group, U of Oklahoma

CAPTURE VELOCITY IN MOT - MINI ZEEMAN SLOWER



MOT BEAM RADIUS R



RESONANCE:

$$0 = \delta_- = \delta + kv + \beta z$$

$$kv = -\delta - \beta z \approx -\delta + \beta R$$

$$v(z) = -\delta/k - \frac{\beta}{k} z$$

MAX v AT $z = -R$: $V_{max} = -\delta/k + \beta R/k$ $\quad \left(\beta = \mu_B B'/\hbar \right)$

Sodium-23: For $\delta = -30\,\text{MHz}$, $B' = 20\,\frac{G}{cm}$, $R = 1\,cm$: $v_c = 34\,m/s$; Molasses $v_c \approx \Gamma/k = 6\,m/s$

- Higher capture velocity than molasses

CHECK ACCELERATION NOT TOO LARGE:

$$a(z) = \frac{dv}{dt} = \frac{dv}{dz}\frac{dz}{dt} = \frac{dv}{dz} v = \left(-\beta/k\right)\left(-\delta/k - \beta z/k\right) = \frac{\beta}{k^2}\left(\delta + \beta z\right)$$

Max $|a|$ when $z = -R$: $a_{max} = \frac{\beta}{k^2}(\delta - \beta R)$

Max possible accel: $a_{lim} = \frac{\Gamma \hbar k}{2M}$; Sodium example: $\left| \frac{a_{max}}{a_{limit}} \right| = 0.06$, Fine

OPTICAL DIPOLE TRAPS (ODT)

LIGHT EXERTS FORCES ON ATOMS/MOLECULES THROUGH 2 MECHANISMS

1. SCATTERING FORCE F_{sc}
   - APPLICATIONS: ZEEMAN SLOWING, OPTICAL MOLASSES, MOTs
2. OPTICAL DIPOLE POTENTIAL (ALSO CALLED AC STARK SHIFT)
   APPLICATIONS: OPTICAL DIPOLE TRAPS, OTHER CONSERVATIVE POTENTIALS

- CLASSICALLY, WE SAW $U_{dipole} = -\frac{1}{2}\langle \vec{E}\cdot\vec{d}\rangle_t$ ← TIME AVG.

- LORENTZ OSCILLATOR: FOR $\vec{E}(t) = E_0\,\hat{x}\cos(\omega t)$, WE FOUND:

$$\vec{d}(t) = -e\,\hat{x}\left[\mathcal{U}\cos(\omega t) - V\sin(\omega t)\right]$$

GIVING: $U_{dipole} = \frac{1}{2}eE_0\langle \mathcal{U}\underbrace{\cos^2(\omega t)}_{\frac{1}{2}} - V\underbrace{\cancel{\sin(\omega t)\cos(\omega t)}}_{0}\rangle_t = \frac{1}{4}eE_0\,\mathcal{U}$

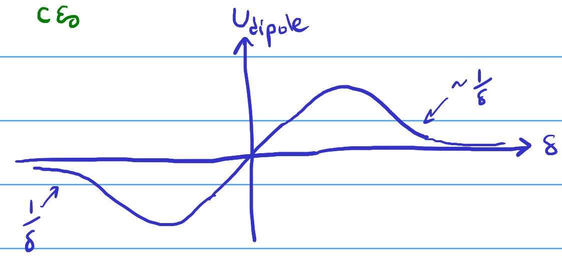
MEANWHILE, $\mathcal{U} \approx \frac{eE_0}{2m\omega_0}\,\frac{\delta}{\delta^2+(\Gamma/2)^2}$ FOR $|\delta| \ll \omega_0$

FINAL LORENTZ OSCILLATOR RESULT IS:

$$U_{dipole} \approx \frac{e^2 E_0^2}{8 m \omega_0} \frac{\delta}{\delta^2 + (\Gamma/2)^2}$$

$$\left\{ E_0^2 = \frac{2I}{c\varepsilon_0} \right.$$

$$= \frac{e^2 I}{4 c \varepsilon_0 m \omega_0} \frac{\delta}{\delta^2 + (\Gamma/2)^2}$$



FOR $|\delta| \gg \Gamma$,

$$U_{dipole} \approx \frac{e^2 I}{4 c \varepsilon_0 m \omega_0} \frac{1}{\delta}$$

PHY 446 SPRING 2020 LECTURE 27 - NANOKELVIN PHYSICS!

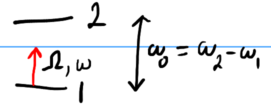
- OPTICAL DIPOLE TRAP (ODT)
- BOSE-EINSTEIN CONDENSATION (BEC)
- DEGENERATE FERMI GASES

MOT : $\vec{F}_{MOT} = -\alpha \vec{v} - \vec{A} \cdot \vec{r}$

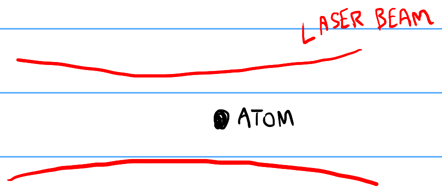
- TRAPS, COOLS to $T \sim T_D \sim 100 \mu K$
- $|\delta| \sim \Gamma \Rightarrow$ LIGHT SCATTERING
- HOW TO REACH LOWER T? 10nK?

→ NEED $|\delta| \gg \Gamma$ TO REDUCE LIGHT SCATTERING

2-LEVEL ATOM



$\delta = \omega - \omega_0$



OPTICAL DIPOLE TRAPS

- DIPOLE POTENTIAL : $U_{dipole} = -\frac{1}{2} \langle \vec{d} \cdot \vec{E} \rangle$
- LARGE DETUNING $|\delta| \gg \Gamma, \Omega$

DIPOLE POTENTIAL (2-LEVEL ATOM)

ELECTRIC FIELD: $\vec{E}(t) = E_0 \hat{x} \cos(\omega t)$

DIPOLE MOMENT (OPERATOR): $\vec{d} = (\hat{d}_x, \hat{d}_y, \hat{d}_z) = -e\vec{r}$ ← POSITION OF EXCITED ELECTRON

$$U_{dipole} = -\frac{1}{2}\langle \vec{d}\cdot\vec{E}\rangle = -\frac{1}{2}\left[\langle \hat{d}_x(t)\rangle E_0 \cos(\omega t)\right]_{\text{time avg.}}$$

↑ VALID WHEN $\Omega \ll |\delta|$

2-LEVEL MODEL: $|\psi\rangle = c_1 e^{-i\omega_1 t}|1\rangle + c_2 e^{-i\omega_2 t}|2\rangle$, $\omega_0 = \omega_2 - \omega_1$, $e^{i\omega_1 t} e^{-i\omega_2 t} = e^{i(\omega_1-\omega_2)t} = e^{-i\omega_0 t}$

DIPOLE MOMENT (QUANTUM) $\langle \hat{d}_x(t)\rangle = \langle \psi(t)|\hat{d}_x|\psi(t)\rangle$ = (PLUG IN...)

$$= \langle 1|\hat{d}_x|2\rangle\left(c_2 c_1^* e^{-i\omega_0 t} + c_1 c_2^* e^{i\omega_0 t}\right)$$

- USING $\langle 1|d_x|1\rangle = 0 = \langle 2|d_x|2\rangle$ BY INVERSION SYMMETRY
- ALSO $\langle 1|d_x|2\rangle = \langle 2|d_x|1\rangle^* = \langle 2|d_x|1\rangle$ (REAL-VALUED)

CHANGE OF VARIABLES

$$e^{i\delta t} e^{i\omega_0 t} = e^{i(\omega-\omega_0)t} e^{i\omega_0 t} = e^{i\omega t}$$

- ROTATING FRAME: $\begin{cases} c_1 = \tilde{c}_1 e^{i\delta t/2} \\ c_2 = \tilde{c}_2 e^{-i\delta t/2} \end{cases}$

- DENSITY MATRIX (ENSEMBLE AVG, FOR SPONTANEOUS EMISSION)

$$\tilde{\rho}_{21} = \langle \tilde{c}_2 \tilde{c}_1^* \rangle_e, \quad \tilde{\rho}_{12} = \langle \tilde{c}_1 \tilde{c}_2^* \rangle_e$$ ← AVG OVER POSSIBLE WAVEFUNCTIONS

- BLOCH VECTOR: $\vec{R} = (u, v, w)$

$$\begin{cases} u = \tilde{\rho}_{12} + \tilde{\rho}_{21} \\ v = -i(\tilde{\rho}_{12} - \tilde{\rho}_{21}) \\ w = \tilde{\rho}_{11} - \tilde{\rho}_{22} \end{cases}$$

DIPOLE MOMENT IN TERMS OF BLOCH VECTOR:

$$\langle d_x(t) \rangle = \langle 1|\hat{d}_x|2\rangle \left[u \cos(\omega t) - v \sin(\omega t) \right]$$

COMPARE TO LORENTZ OSCILLATOR:

$$d_x(t) = -e \left[U \cos(\omega t) - V \sin(\omega t) \right]$$

DIPOLE POTENTIAL

$$U_{dipole} = \frac{-1}{2}\left[\langle \hat{d}_x(t)\rangle E_0 \cos(\omega t)\right]_{\text{time avg.}} = \frac{-1}{2}\langle 1|d_x|2\rangle E_0\left[u \underbrace{\cos^2\omega t}_{1/2} - v\,\cancel{\underbrace{\sin(\omega t)\cos(\omega t)}_{0}}\right]_t$$

$$= \frac{-\langle 1|d_x|2\rangle E_0}{4} u$$

STEADY-STATE BLOCH VECTOR $\quad u_{ss} = \frac{\Omega\delta}{\delta^2 + \Omega^2/2 + \Gamma^2/4} \approx \frac{\Omega}{\delta}$ FOR $|\delta| \gg \Omega, \Gamma$

RECALL: $\hbar\Omega \equiv -E_0\langle 1|\hat{d}_x|2\rangle \Rightarrow u_{ss} \approx \frac{-E_0\langle 1|d_x|2\rangle}{\hbar\delta}$

$$U_{dipole} \approx \frac{|\langle 1|d_x|2\rangle|^2 E_0^2}{4\hbar\delta}$$

$$= \frac{|\langle 1|d_x|2\rangle|^2 I}{2\varepsilon_0 c\,\hbar\delta}$$

COMPARE TO LORENTZ OSCILLATOR

$$U_{dipole} \approx \frac{e^2 I}{4c\varepsilon_0 m_e\omega_0\delta}$$

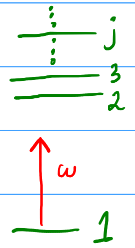
QUANTUM VS. CLASSICAL

RATIO: $\dfrac{U_{QUANTUM}}{U_{classical}} = \dfrac{2 m_e \omega_0}{\hbar e^2} |\langle 1 | d_x | 2 \rangle|^2 = f_{12}$ "OSCILLATOR STRENGTH"

- WHEN $f_{12} = 1$, LORENTZ OSCILLATOR AND 2-LEVEL MODEL AGREE
- OFTEN, $f_{12} \sim 1$, BUT NOT ALWAYS

GENERALIZE TO MULTILEVEL ATOMS

$$U_{dipole} \approx \frac{I}{2\varepsilon_0 c \hbar} \sum_j \frac{|\langle 1 | \hat{d}_x | j \rangle|^2}{\omega - \omega_{j1}}$$



WHERE $\omega_{j1} = \omega_j - \omega_1$

LIGHT SCATTERING IN DIPOLE TRAPS

$$R_{sc} = \Gamma \rho_{22} = \frac{\Gamma}{2} \frac{S}{1+S+(2\delta/\Gamma)^2} \approx \frac{\Gamma}{2} \frac{S}{(2\delta/\Gamma)^2}$$

USING

AND $|\delta| \gg \Gamma, \Omega$

$$S = 2\Omega^2/\Gamma^2 = \frac{I}{I_{SAT}}$$

$$= \left(\frac{\Gamma^3}{8 I_{SAT}}\right) \frac{I}{\delta^2}$$

MINIMIZING R_{sc}:

$$U_{dipole} \propto \frac{I}{\delta}$$

$$R_{sc} \propto \frac{I}{\delta^2} \propto \frac{U_{dipole}}{\delta}$$

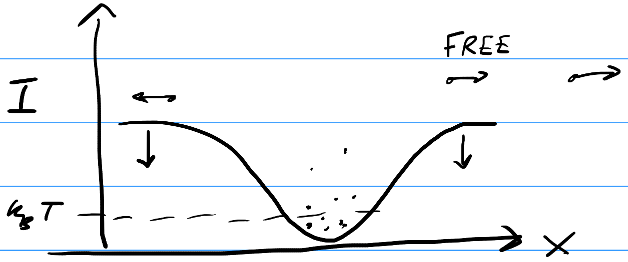
- CAN MAKE R_{sc} ARB. SMALL BY MAKING $|\delta|$ LARGE & I LARGE

EVAPORATIVE COOLING

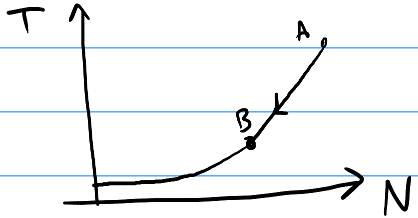
- REACH $T \sim nK$

REDUCE I

$\Rightarrow$

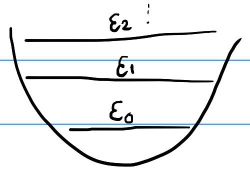


- LOWER POTENTIAL SLOWLY → RETHERMALIZE

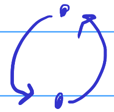


$\sim 10^{-9} K$

QUANTUM STATISTICS



ENERGY LEVELS IN A TRAP



$$\psi_2 \rightarrow e^{i\theta}\psi_2$$

\# ATOMS

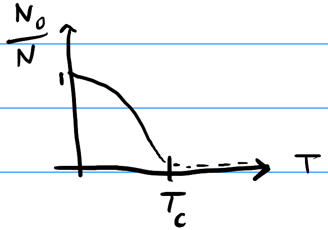
$$N = \sum_i f(\varepsilon_i, \mu) \qquad \beta = \frac{1}{k_B T}$$

BOSE-EINSTEIN STATISTICS: $f_{BE}(\varepsilon) = \dfrac{1}{e^{\beta(\varepsilon-\mu)} - 1}$ BOSONS: $\theta = 0$, $S = $ INTEGER

FERMI-DIRAC STATISTICS: $f_{FD}(\varepsilon) = \dfrac{1}{e^{\beta(\varepsilon-\mu)} + 1}$ FERMIONS: $\theta = \pi$, $S = \dfrac{\text{INT}}{2}$

BOSE-EINSTEIN CONDENSATION - OVERVIEW

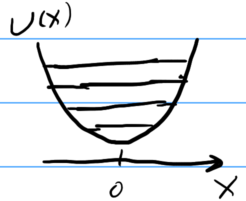
For $T < T_c$, $N_0 = f(\epsilon_0) \to$ LARGE $\sim N$

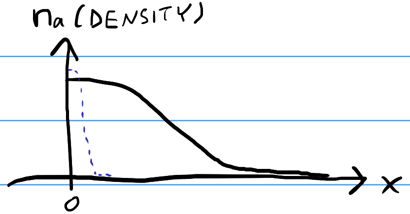


$$\frac{N_0}{N} \approx 1-\left(\frac{T}{T_c}\right)^{3/2} \qquad (T<T_c)$$

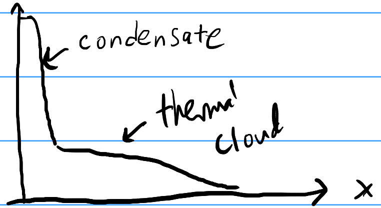
N_0 - "CONDENSATE"

OBSERVATION:





$\Rightarrow$



EXTRA-DERIVATION

DIPOLE MOMENT (QUANTUM)

$$\langle d_x(t)\rangle = \langle\Psi(t)|d_x|\Psi(t)\rangle = \left[c_1^* e^{i\omega_1 t}\langle 1| + c_2^* e^{i\omega_2 t}\langle 2|\right] d_x \left[c_1 e^{-i\omega_1 t}|1\rangle + c_2 e^{-i\omega_2 t}|2\rangle\right]$$

$$= c_1^* c_2 e^{-i\omega_0 t}\langle 1|d_x|2\rangle + c_2^* c_1 e^{i\omega_0 t}\langle 2|d_x|1\rangle$$

- USING $\langle 1|d_x|1\rangle = 0 = \langle 2|d_x|2\rangle$ BY INVERSION SYMMETRY
- ALSO, $\langle 1|d_x|2\rangle = \langle 2|d_x|1\rangle^* = \langle 2|d_x|1\rangle$ (REAL-VALUED)

$$= \langle 1|d_x|2\rangle\left(c_2 c_1^* e^{-i\omega_0 t} + c_1 c_2^* e^{i\omega_0 t}\right)$$

ROTATING FRAME TRANSFORMATION: $c_1 = \tilde{c}_1 e^{i\delta t/2}$, $c_2 = \tilde{c}_2 e^{-i\delta t/2}$ $(\delta = \omega - \omega_0)$

$$c_2 c_1^* = \tilde{c}_2 \tilde{c}_1^* e^{-i\delta t} = \tilde{c}_2 \tilde{c}_1^* e^{i(\omega_0 - \omega)t}$$

$$c_1 c_2^* = \tilde{c}_1 \tilde{c}_2^* e^{i\delta t} = \tilde{c}_1 \tilde{c}_2^* e^{i(\omega - \omega_0)t}$$

DIPOLE MOMENT IN "ROTATING FRAME" VARIABLES:

$$\langle d_x(t)\rangle = \langle 1|d_x|2\rangle\left(\tilde{c}_2 \tilde{c}_1^* e^{-i\omega t} + \tilde{c}_1 \tilde{c}_2^* e^{i\omega t}\right)$$

ENSEMBLE AVG (FOR SPONTANEOUS EMISSION): $\tilde{\rho}_{21} = \langle \tilde{c}_2 \tilde{c}_1^* \rangle_e$, $\tilde{\rho}_{12} = \langle \tilde{c}_1 \tilde{c}_2^* \rangle_e$

$$\langle d_x(t)\rangle = \langle 1|d_x|2\rangle\left(\tilde{\rho}_{21} e^{-i\omega t} + \tilde{\rho}_{12} e^{i\omega t}\right)$$

EULER'S FORMULA: $e^{i\theta} = \cos\theta + i\sin\theta$

$$\langle d_x(t)\rangle = \langle 1|d_x|2\rangle\left[\tilde{\rho}_{21}(\cos\omega t - i\sin\omega t) + \tilde{\rho}_{12}(\cos\omega t + i\sin\omega t)\right] = \langle 1|d_x|2\rangle\left[(\tilde{\rho}_{21}+\tilde{\rho}_{12})\cos\omega t + i(\tilde{\rho}_{12}-\tilde{\rho}_{21})\sin\omega t\right]$$

BLOCH VECTOR:

$$u = \tilde{\rho}_{12} + \tilde{\rho}_{21}$$

$$v = -i(\tilde{\rho}_{12} - \tilde{\rho}_{21})$$

$$w = \tilde{\rho}_{11} - \tilde{\rho}_{22}$$

DIPOLE MOMENT IN TERMS OF BLOCH VECTOR:

$$\langle d_x(t)\rangle = \langle 1|d_x|2\rangle\left[u\cos(\omega t) - v\sin(\omega t)\right]$$